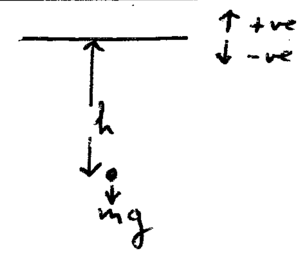
Mechanics Examples Sheet 6 — Solutions

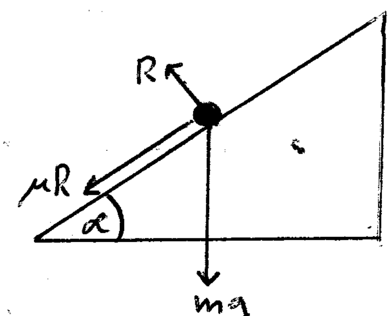


1. As the particle falls under a constant acceleration, g, we can use the formula

$$v^2 = u^2 + 2gh \ , \quad u=0 \text{ as particle falls from rest.}$$

$$\left.\begin{aligned} K.E. &= \tfrac{1}{2}mv^2 = mgh \\ P.E &= -mgh \end{aligned}\right\} K.E.+P.E. = 0 \ , \text{ constant.}$$

2. Let R be the normal reaction.
Using N2:



$\perp$ to plane: $R - mg\cos\alpha = 0$

↑↑ to plane: $\mu R + mg\sin\alpha = ma$

where a is the acceleration of the particle down the plane. On eliminating R:

$$\underline{a = g(\sin\alpha + \mu\cos\alpha)}$$

The particle experiences a constant acceleration, so we can use the formula

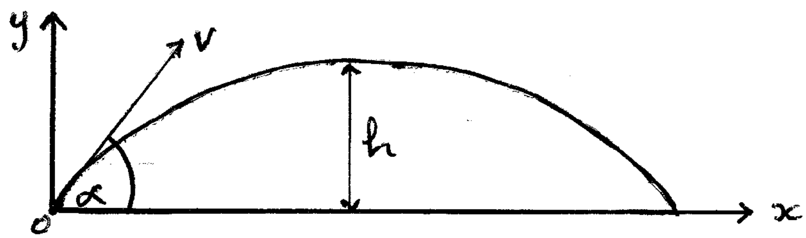
$$v^2 = u^2 - 2aS \quad (\text{-ve due to direction down the slope})$$

When the particle comes to rest, $v=0$ $\therefore$ $\underline{S = \dfrac{1}{2}\dfrac{u^2}{a}}$

$$\begin{aligned} W.D. &= \Delta K.E. + \Delta P.E. \\ &= \tfrac{1}{2}mu^2 - mgS\sin\alpha \\ &= \tfrac{1}{2}mu^2\left(1 - \frac{g\sin\alpha}{a}\right) \\ &= \tfrac{1}{2}mu^2\left(1 - \frac{\sin\alpha}{\sin\alpha + \mu\cos\alpha}\right) \\ &= \underline{\tfrac{1}{2}mu^2\left(\frac{\mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}\right)} \end{aligned}$$

$$W.D. = \text{frictional force} \times \text{distance}$$
$$= \mu R \times S$$
$$= \mu mg\cos\alpha \times \frac{1}{2}\frac{u^2}{a}$$
$$= \underline{\underline{\frac{1}{2}mu^2\left(\frac{\mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}\right)}}$$

3.



Initially: $K.E. = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

$$= \frac{1}{2}m(v^2\cos^2\alpha + v^2\sin^2\alpha)$$
$$= \frac{1}{2}mv^2$$
$$P.E = 0$$

Finally: $K.E = \frac{1}{2}m\dot{x}^2$

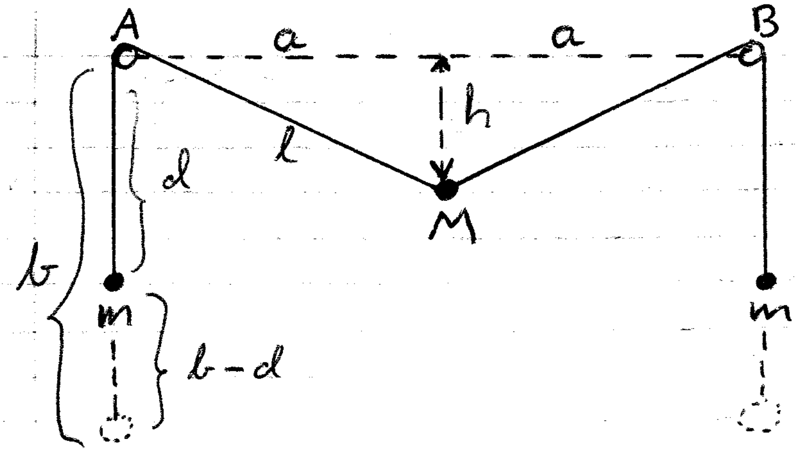
$$= \frac{1}{2}mv^2\cos^2\alpha$$ (N.B. horizontal velocity is constant throughout motion)
$$P.E = mgh$$

Using conservation of energy:

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2\cos^2\alpha + mgh$$

$$\Rightarrow \quad \underline{h = \frac{1}{2}v^2\frac{\sin^2\alpha}{g}}$$

4.



$M < 2m$

Initially m is a distance b bellow A and B.

M falls vertically by symmetry. When M drops by a distance h, m rises a distance $b-d$. Thus

$$a + b = l + d$$

or

$$b - d = l - a.$$

But

$$l = \sqrt{a^2 + h^2}$$

$$\therefore \quad b - d = \sqrt{a^2+h^2} - a.$$

Now,

$$P.E._{in} = 0 \quad , \quad K.E._{in} = 0$$

$$P.E._{fin} = -Mgh + 2mg(b-d) \quad , \quad K.E._{fin} = 0$$

Therefore, by conservation of energy

$$-Mgh + 2mg(b-d) = 0$$

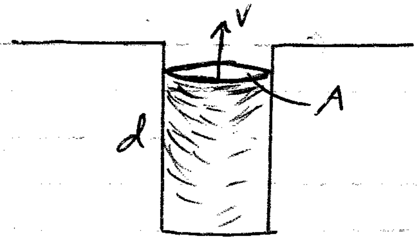
$$\Rightarrow \quad Mh = 2m(\sqrt{a^2+h^2} - a)$$

$$\Rightarrow \quad (Mh + 2am)^2 = 4m^2(a^2+h^2)$$

$$\Rightarrow \quad (4m^2 - M^2)h^2 = 4amMh$$

$$\Rightarrow \quad h = \underline{\underline{\frac{4amM}{4m^2 - M^2}}}$$

5. In one second, say, the volume of water raised from a depth d is $A(V/1) = AV$, so the total final K.E. of this water is



$$K.E._{fin} = \frac{1}{2}(Av\rho)v^2$$
$$= \frac{1}{2}A\rho v^3$$

The P.E. is mass $\times g \times d = (\rho A V) g d$. ($\rho$ = mass/vol)

In one second, the work done by the pump is

$$WD = \text{change in total energy}$$
$$= \frac{1}{2}A\rho v^3 + \rho A v g d.$$

But, in one second WD = power.